Here I try to show that **all valid Septoku boards must have unique symbols in the group made up of circle centers.** In other words, I will show that it is impossible for there to be two (or more) circle centers with the same symbol. I will do this by showing that placement of duplicate symbols in circle centers leads inevitably to placement of symbols in a manner inconsistent with the rules of Septoku.

We'll call the center of a gray-circle group the "Circle Group Center" or 'CGC' for short. Suppose there are two CGCs that share the same number symbol. If that's the case, then there is only 1 configuration where those two CGCs are not in the same row. Thus if I can show that that configuration forces an inconsistent placement of numbers, I will have shown that it is not possible for there to be two CGCs that share the same number symbol.

We'll use the number 7 as our number to be duplicated. The only possible configuration looks like this:

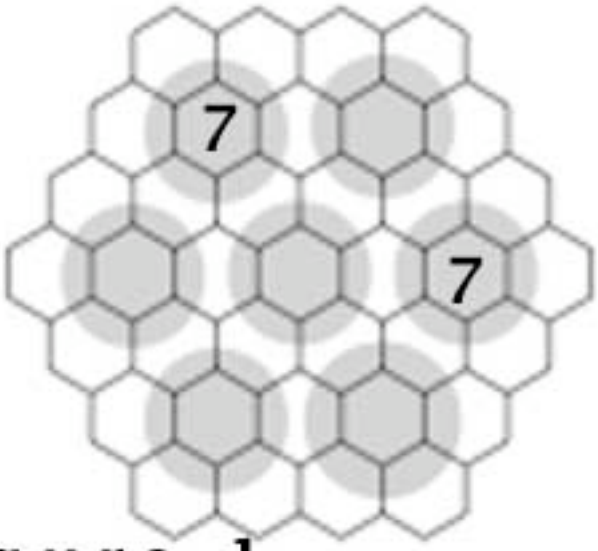

figure 1

Any other placement of two 7s in two different CGCs will either be a rotation of figure 1 (and hence for the purposes of this proof will count as the same configuration) or will have the two 7s in the same row as each other, which is not allowed.

(Again, notice that rotating the board will make no difference to this proof. Thus although I may refer to various areas as the "top right" or "lower left", we could rotate the board and it would not make a difference.)

Now let us cross out all the hexagons where 7 cannot appear, due to its being in the same row or circle group as one of the two 7s. This gives us figure 2:

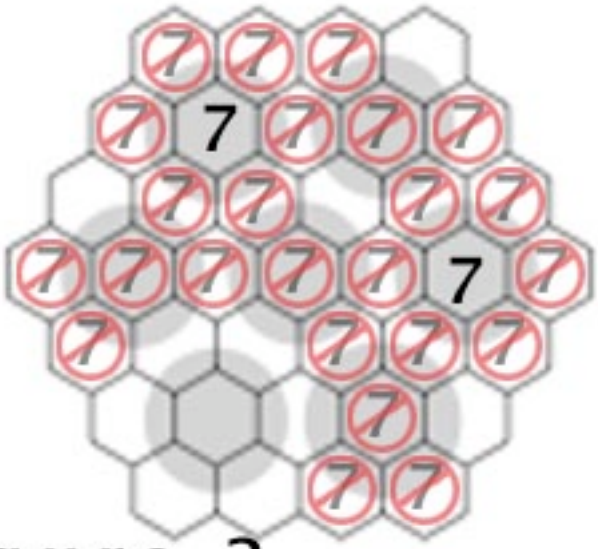

figure 2

(Interestingly, there are exactly twenty-four places where sevens cannot be. 24 is clearly a magic number in Septoku!)

Now notice that the gray circle group which is in the *center* of the board does not contain a 7 and has only two places where the 7 could be: in the lower left hexagon of that circle group, or in the upper right hexagon. If we were to place the 7 in the lower left hexagon, then there is no consistent place for a 7 to go in the grey circle group at the top right. (See figure 3).

figure 3

Thus, when placing the 7 in the gray circle group that is in the center of the board the only place that the 7 can go is in the upper right hexagon (where it overlaps with the gray circle group in the upper right part of the board). This will rule out a few more places where 7s can go:

figure 4

Now there are three gray circle groups that do not have a 7 in them. They are: the leftmost circle group (LCG), the bottom left circle group (BLCG), and the bottom right circle group (BRCG). But there is no consistent place where these three circle groups could have a 7. LCG and BRCG each have only one place where a 7 could be. But if we fill in both those places, then BLCG will have two 7s in it, which it is not allowed to have:

figure 5

To Summarize:

(1) There is only one configuration (at least if we count rotated configurations as the same configuration) where two duplicate numbers can appear in different CGCs but where they are not in the same row as each other.
(2) This placement rules out a number of hexagons as places where 7s can appear, and thus allows only two hexagons where the gray circle group in the center of the board can have a 7: the top right of that circle group or the bottom left of that circle group.
    A. If we place the central gray circle group's 7 in its bottom left hexagon, then there is no place to put the 7 in the gray circle group that is at the top right of the board. Thus the 7 cannot go here.
    B. Thus the 7 for the gray circle group in the center of the board must be placed in the top right hexagon (directly between the other two 7's, though not in a row with them.)
(3) But this placement rules out any consistent placement of 7s in the leftmost, bottom-left, and bottom-right gray circle groups.
(4) Thus there is no consistent way to have duplicate numbers in any of the center hexagons of the gray circle groups.

ADDENDUM:
One might think that I have only shown that it is impossible for there to be *exactly* 2 duplicate numbers in any two CGCs. Is it still possible for there to be *more than* 2 duplicate numbers? Four or more duplicates will necessitate that some of them are in the same row as each other, which is not allowed. What about three duplicates? You *could* place three 7s in three different CGCs without any of them being in the same row:

figure 6

But then we can see that there are three circle groups (including the one in the center) which lack 7s but cannot have a 7 due to a row conflict with one of the three 7's placed initially. Thus three CGC duplicates is not possible on a valid Septoku board. Thus having more than 2 duplicate numbers in different CGCs won't work any more than having exactly 2 duplicates would.

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